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Magnetohydrodynamic theory and its role in plasma modeling for tokamak magnetic control

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Abstract
Magnetohydrodynamics describes the fundamental behavior of magnetically confined plasma. In this theory, plasma in considered as a single fuild, that is, there is no distinction between the different particles that make up plasma.Plasma is completely described by the density of ρ and velocity vector of the fuild. Plasma is the fourth state of matter.Plasma is an ionized gas in which electrons are separated from the nuclei of atoms and positively and negatively charged particles move in depepentdently.There fore,because plasma particles are charged, they conduct electric current and interact with magnetic fields.Tokamak is derived from generalized Russian vocabulary: Toroidalnaya KAmera ee MAgnitaya Katushka.It means a toroidal chamber with magnetic coils. As its names suggests a tokamak is a magnetic confinement derive with a toroidal geometry.

Key Words:
Magnetohydrodynamics,Plasma,Tokamak,Magnetic control,Maxwell,Toroidal geometry

۱. Introduction:

As mentioned in magnetohydrodynamic theory, plasma is compeletly described by the density and velocity vector of the fuild. The basic laws that link these committees are the law of survival of the fittest:

$$\frac{\partial}{\partial t}\rho + \nabla.(\rho v) = ۰$$

(۱)

and Newton's law for a very small element of plasma:

$$\rho \frac{d}{dt} v = J \times B - \nabla p$$

(۲)

Where J is the field current density,B is the magnetic induction field and p is the kinetic pressure inside the plasma.

The coupling between the electromagnetic field and the plasma is represented by the Lorentz force term in equation ۲. [۱,۲,۳]

In addition, electromagnetic fields must obey Maxwell's which are:

$$\nabla. B = ۰$$

Gauss's law(۳)

$$\nabla \times H = J$$

Amper's law(٤)

$$\nabla \times E = -\frac{\partial}{\partial t} B$$

Faraday's law(٥)

Equation three is Gauss's law for magnetic field induction. Equation four is Amper's law,which gives the relationship between current density and magnetic field strength H and the equation is Faraday's fifth law, where E is the electric field.

In Amper's law, the time derivative of the displacement to the electric field (usually denoted by D) is neglected. This is consistent with neglecting capacitive noise effects.This assumption is consistent with the time scale of the phenomena involved:

These relationships ultimately complete the set of ideal magnetohydrodynamic equations:

$$B=\mu.H \tag{٦}$$

$$\eta J=E+V\times B \tag{٧}$$

The previous equations can be summarized in a table as follows:

$$\nabla.B = ٠ \qquad \frac{\partial}{\partial t}\rho + \nabla.(\rho V) = ٠$$

$$\nabla \times B = \mu.J \qquad \rho\frac{d}{dt}V = J \times B - \nabla P$$

$$\nabla \times E = -\frac{\partial}{\partial t}B \qquad E+V\times B = \eta J$$

٢.Magnetohydrodynamic equations in toroidal geometry with axial symmetry:

This section refers to the polar flux function. Because a tokamak device with axial symmetry,it is better to write the magnetohydrodynamic equations in the three- dimensional cylindrical coordinates (r,φ, z),Where r is the radial coordinate, φ is the polar angle and z is the height.i_r, i_φ *and* i_z are unit-axis vectors. The $\Gamma_{(r)}$of the circumference created by rotating point r around the r=o.

S(r) represents the surface that contains the $\Gamma_{(r)}$ as an edge. The components of a typical A along the unit vectors are given by A_r, A_φ *and* A_z *respectively* ,Which can be written as: A:$A_r i_r + A_\varphi i_\varphi + A_z i_z$.

A any point, the direction parallel to the unit vector i_φ is called the torus direction.While the plane perpendicular to this direction is called the polar plane; this plane is characterized by a constant toroidal angle φ.

In addition, due to the axially symmetric toroidal geometry of the tokamak device,it can be assumed that not all the quantities present depend on the toroidal angle.There fore referring back to the type vector A, it may be assumed that: $\frac{\partial}{\partial \varphi}A = ٠$.

Using the assumption of axial symmetry, Gauss's iaw (Equations٣) is written in cylindrical as follows:

$$\frac{۱}{r}\frac{\partial}{\partial r}rB_r + \frac{\partial}{\partial z}B_z = ۰$$

(۸)

In this case, it is appropriate to introduce the polar flux function as follows:

$$\psi(r) = \frac{۱}{۲\pi}\int_{s(r)} B.ds$$

(۹)

Because the surface integral in equation (۹) does not depend on the specifics s_r and only on its edge depends $\Gamma_{(r)}$. By choosing s_r perpendicular to any point of i_z case in figure(۱).we will have:

$$\psi(r) = \frac{۱}{۲\pi}\int_۰^r \int_۰^{۲\pi} B_z(\rho,z)\rho d\rho d\varphi = \int_۰^r \rho B_z(\rho,z)d\rho$$

(۱۰)

By differentiating equation (۱۰) with respect to r, we have :$\frac{\partial}{\partial r}\psi = rB_z$.

By differentiating the same equation with respect to ۲ and considering equation (۸), we conclude:$\frac{\partial}{\partial z}\psi = -rB_r$.

There fore, the polar flux function and the magnetic induction field are related through the following equations:

$$B_r = \frac{۱}{r}\frac{\partial}{\partial z}\psi$$

(۱۱)

$$B_z = -\frac{۱}{r}\frac{\partial}{\partial r}\psi$$

(۱۲)

Equations (۱۱) and (۱۲) can be written in the following vector form, taking into account that $\nabla\varphi = r^{-۱}i_\varphi$ *is* equal to one:

$$B_p = B_r i_r + B_z i_z = \nabla\psi \times \nabla\varphi$$

(۱۳)

B_p is the mage of the magnetic induction field on the polar plane. Note that the existence of the scalar function ψ,Which applies in equations (١١) and (١٢) is simply a result of the non-divergence of the magnetic induction field.

There fore, by applying the divergence term in Ampere’s law (Equation) (considering that the divergence of a rotation is zero),it is easily shown that the current density vector J does not diverge. Therefore, there will be scalar function f that satisfies the following relations:

$$J_r = -\frac{١}{r}\frac{\partial}{\partial z}f$$

(١٤)

$$J_z = \frac{١}{r}\frac{\partial}{\partial r}f$$

(١٥)

Combining Ampere’s law (Equation ٤) with the basic relation (٦) and assuming axial symmetry in cylindrical coordinates the following relations are obtained:

$$-\frac{\partial}{\partial z}B_\varphi = \mu.J_r$$

(١٦)

$$\frac{\partial}{\partial z}B_r - \frac{\partial}{\partial r}B_z = \mu.J_\varphi$$

(١٧)

$$\frac{١}{r}\frac{\partial}{\partial r}rB_\varphi = \mu.J_z$$

(١٨)

Combining equations (١٤) and (١٥) with equations (١٦) and (١٨) yields:$B_\varphi = \mu.\frac{f}{r}$.

If we assume F(r,z)=⬚⬚$(f(r,z)$,the toroidal component of the magnetic induction field is written as:

$$B_\varphi = F\nabla\varphi.i_\varphi$$

(١٩)

Finally, the magnetic induction field can be expressed as a combination of two scalar functions ψ and f:

$$B = \nabla\psi \times \nabla\varphi + F\nabla\varphi$$

(٢٠)

The first term on the right side of equation (۲۰) is the image of the magnetic induction field on the polar plane (polar magnetic field. While the second term is the toroidal component magnetic induction field). By substituting equation (۲۰) into Ampere’s law (equation ٤), we obtain:

$J=\mu_{\cdot}^{-۱}\nabla \times (\nabla\psi \times \nabla\varphi) + F\nabla\varphi$

$=-\mu_{\cdot}^{-۱}\Delta^{*}\nabla\varphi + \mu_{\cdot}^{۱-}\nabla F \times \nabla\varphi$

(۲۱)

The Δ^{*} operator is the elliptic derivative and is defined by this relation: $\Delta^{*}X = r^{-۲}\nabla.(r^{-۲}\nabla X)$=$r\frac{\partial}{\partial r}(\frac{۱}{r}\frac{\partial}{\partial r}X)+\frac{\partial^{۲}}{\partial z^{۲}}X$.

The representation of equation (۲۱) along the toroidal direction gives this relationship:

$\Delta^{*}\psi = -\mu_{\cdot} rJ_{\varphi}$

(۲۲)

Where J_{φ}, *is the toroidal current density.*

Another useful relationship can be expressed between the toroidal component of the electric field and the time derivative of the polar flux function.

Starting from Faraday’s law (equation ٥) and applying the Kelvin-Stokes theorem, we have : $\oint_{\Gamma(r)} E.dL = -\partial/\partial t \int_{s(r)} B.ds = -۲\pi\frac{\partial}{\partial t}\psi$

Fromit, this relationship can be easily obtained:

$E_{\varphi} = -\frac{۱}{r}\frac{\partial}{\partial t}\psi$

(۲۳)

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Conclusion:

Magnetohydrodynamics describes the fundamental behavior of magnetically confined plasma, Where the plasma is considered as a single fluid, i.e. there is no distinction between the different particles that make up the plasma. because the tokamak is a toroidal device with axial symmetry, it is better to write the magnetohydrodynamic, equations in a three dimensional cylindrical coordinate system (r,φ, z), in which case is r=٠, we consider the tokamak's rotational axis. When the magnetohydrodynamic equations in a three-dimensional cylindrical coordinate system (r,φ, z) there are changes in these equations with the three-dimensional cylindrical coordinate system, Which are necessary and essential for the tokamak, Which has a toroidal geometry with axial symmetry.

Referencess:

١.Freidberg J.P.(١٩٨٧)Ideal Magnetohydrodynamics. Plenum press, New York.
٢.Shafranov V.D.(١٩٦٦) Plasma equilibrium in a magnetic field. Reviews of plasma physics ٢:١٠٣-١٥١.
٣.Friedberg J.P.(١٩٨٢)Ideal magnetohydrodynamic theory of magnetic fusion systems.Review of Modern physics ٥٤:٩٠٢.